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Threshold radar reflectivity for drizzling clouds

Yangang Liu,[1] Bart Geerts,[2] Mark Miller,[2] Peter Daum,[1] and Robert McGraw[1]

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[1] Empirical studies have suggested the existence of a threshold radar reflectivity between nonprecipitating and precipitating clouds; however, there has been neither a rigorous theoretical basis for the threshold reflectivity nor a sound explanation as to why empirically determined threshold reflectivities differ among studies. Here we present a theory for the threshold reflectivity by relating it to the autoconversion process. This theory not only demonstrates the sharp transition from cloud to rain when the radar reflectivity exceeds some value (threshold reflectivity) but also reveals that the threshold reflectivity is an increasing function of the cloud droplet concentration. The dependence of threshold reflectivity on droplet concentration suggests that the differences in empirically determined threshold reflectivity arise from the differences in droplet concentration. The favorable agreement with measurements collected over a wide range of conditions further provides observational support for the theoretical formulation. The results have many potential applications, especially to remote sensing of cloud properties and studies of the second aerosol indirect effect. **Citation:** Liu, Y., B. Geerts, M. Miller, P. Daum, and R. McGraw (2008), Threshold radar reflectivity for drizzling clouds, *Geophys. Res. Lett.*, *35*, L03807, doi:10.1029/2007GL031201.

1. Introduction

[2] Millimeter-wave cloud radars have found increasing application in remote sensing of cloud properties [*Clothiaux et al.*, 1995; *Moran et al.*, 1998; *Galloway et al.*, 1999; *Stephens et al.*, 2002; *Kollias et al.*, 2005]. Cloud radars operating at millimeter wavelengths generally receive greater echo intensity from cloud droplets than conventional precipitation radars operating at longer wavelengths (e.g., centimeters) because radar backscattering of spherical water droplets/drops decreases with increasing radar wavelength. In drizzle-free clouds, the radar reflectivity is a good measure of the cloud liquid water content [*Wang and Geerts*, 2003]. However, in precipitating clouds, the radar reflectivity may be dominated by the presence of few drizzle-sized drops [*Fox and Illingworth*, 1997; *Vali et al.*, 1998]. Thus, different algorithms are needed for retrieving cloud properties of nonprecipitating and precipitating clouds, and distinguishing between nonprecipitating and precipitating clouds is critical for use of remote sensing techniques. Identification of drizzle occurrence is also essential for studying the non-adiabatic behavior of clouds [*Chin et al.*, 2000].

[3] Several studies have proposed a threshold in radar reflectivity as the basis of discrimination between non-precipitating and precipitating clouds. However, there appears to be no physically sound approach for specification of the threshold reflectivity, and a wide range of values has been used. For example, *Chin et al.* [2000] used the threshold of −15 dBZ as an indication of drizzle occurrence in their microphysical retrieval of continental stratiform clouds. *Kato et al.* [2001] used −20 dBZ in their retrieval of continental stratus. *Matrosov et al.* [2004] found a gradual deterioration of the liquid water content (L) retrieved from radar reflectivity when reflectivity threshold set for the retrieval increases, and at high reflectivity values an ambiguity exists between clouds with high L and those with drizzle. *Kogan et al.* [2005] used −17 dBZ as the reflectivity threshold to partition their observations into non-precipitating and precipitating clouds. They also examined the influence of varying the threshold between −20 and −15 dBZ on their results.

[4] Heretofore, the issue of the threshold reflectivity that separates precipitating from nonprecipitating clouds has been examined primarily by empirical analysis of observational data, and a quantitative theoretical investigation is lacking. The cause of the difference between various empirically determined threshold reflectivities remains largely unknown. In this contribution the threshold reflectivity is derived from first principles and is related to physically relevant cloud properties such as the cloud droplet number concentration (N). Empirical support for this new formulation is provided by comparison to observations.

2. Threshold Function

2.1. Theoretical Expression

[5] Warm rain starts with the autoconversion process whereby cloud droplets grow into embryonic drizzle drops. In a series of publications [*Liu et al.*, 2004, 2005, 2006, 2007], we have theoretically demonstrated that the cloud-to-rain transition behavior of the autoconversion process can be described by the general threshold function given by

$$T = \left[\frac{\int_{r_c}^{\infty} r^6 n(r)dr}{\int_{0}^{\infty} r^6 n(r)dr}\right]\left[\frac{\int_{r_c}^{\infty} r^\delta n(r)dr}{\int_{0}^{\infty} r^\delta n(r)dr}\right], \quad (1)$$

where the exponent 6 in the first square bracket arises from the collection process (collision and coalescence), and the exponent δ in the second square bracket denotes the order of the power moment of the cloud droplet size distribution n(r) in question. The critical radius r_c, beyond which the

[1]Brookhaven National Laboratory, Upton, New York, USA.
[2]Department of Atmospheric Sciences, University of Wyoming, Laramie, Wyoming, USA.

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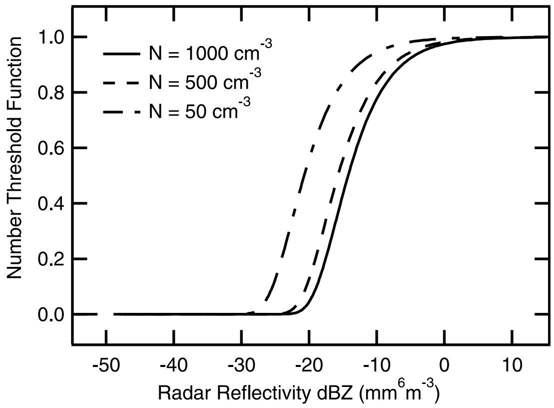

Figure 1. Dependence of the number threshold function on the cloud radar reflectivity. The different curves represent results calculated from the analytical threshold function at different cloud droplet concentrations (N).

collection process become dominant, corresponds to the kinetic potential barrier of the droplet population and is a function of N and L (see *McGraw and Liu* [2003, 2004] and *Liu et al.* [2004] for the exact definition of r_c and more discussion). When $\delta = 0$, 3, and 6, equation (1) describes the transition behavior of N, L, and radar reflectivity, respectively.

[6] It is evident from equation (1) that the threshold function is determined by the contribution from cloud droplets with radii larger than the critical radius relative to that from all the droplets, which depends on the value of the critical radius, and the spectral shape of the cloud droplet size distribution. For the general Weibull droplet size distribution [*Liu and Hallett*, 1997; *Liu and Daum*, 2000],

$$n(r) = \frac{qN}{r_0^q} r^{q-1} \exp\left[-\left(\frac{r}{r_0}\right)^q\right], \tag{2}$$

equation (1) becomes

$$T = \gamma\left(\frac{6+q}{q}, x_{cq}\right)\gamma\left(\frac{\delta+q}{q}, x_{cq}\right), \tag{3a}$$

$$x_{cq} = \left(\frac{r_c}{r_0}\right)^q = \Gamma^{q/3}\left(\frac{3+q}{q}\right)x_c^{q/3}, \tag{3b}$$

$$x_c = 9.7 \times 10^{-17} N^{3/2} L^{-2}, \tag{3c}$$

where q is a parameter that depends on the spectral shape of the cloud droplet size distribution; r_0 is the mode radius; x_c is the ratio of the critical to mean masses of the droplet population; Γ and γ represent the complete and incomplete Gamma functions, respectively. When $\delta = 0$, T becomes the number threshold function, and is given by

$$T_N = \gamma\left(\frac{6+q}{q}, x_{cq}\right)\gamma\left(1, x_{cq}\right) \tag{3d}$$

[7] For radar applications, it is desirable to express the threshold function, or, x_c, in terms of the radar reflectivity factor Z such that

$$x_c = 9.7 \times 10^{-17} 64\left(\frac{3}{4\pi\rho_w}\right)^2 \Gamma\left(\frac{6+q}{q}\right)\Gamma^{-2}\left(\frac{3+q}{q}\right)N^{1/2}Z^{-1}, \tag{4a}$$

$$Z = 64\int r^6 n(r)dr. \tag{4b}$$

[8] Substitution of equations (4a) and (4b) into equation (3d) gives the equation that quantifies the dependence of the number threshold function on the radar reflectivity as well as N and q. Utilizing the typical droplet size distribution with q = 3 yields a simpler number threshold function given by

$$T_N = \gamma(3, x_c)\gamma(1, x_c), \tag{5a}$$

$$x_c = 7.1 \times 10^{-16} N^{1/2} Z^{-1}. \tag{5b}$$

[9] Figure 1 shows the dependence of the number threshold function calculated from equations (5a) and (5b) on radar reflectivity for different values of N (N = 50 cm^{-3}, N = 500 cm^{-3}, and N = 1000 cm^{-3}). Note that the change in the threshold function with reflectivity is like a phase transition: first the threshold function changes little when radar reflectivity increases; when the reflectivity reaches a certain value it increases sharply, and then remains almost unchanged as the radar reflectivity further increases. This behavior provides theoretical support for the common practice of using a threshold reflectivity to separate precipitating from non-precipitating clouds, and can be better understood by further examining equations (5a) and (5b). Equation (5a) indicates that the number threshold function first gradually increases with decreasing x_c and abruptly levels off at one when $x_c \leq 1$ (not shown here). Equation (5b) further indicates that for a given N, x_c decreases when the radar reflectivity increases (because of the combined increase in L and decrease in the critical radius). Also note that, the "phase-transition point" shifts to the right with increasing N, suggesting that the threshold reflectivity is not a constant as commonly assumed, but increases with increasing N. This important point will be further examined in Section 3.

2.2. Comparison with Observations

[10] *Wang and Geerts* [2003] proposed an approach to empirically examine the transition from non-precipitating to precipitating clouds by determining the occurrence probability of drizzle-sized drops for a given radar reflectivity value. Drizzle was deemed present if the particle count of the Particle Measurement Systems 2D-C probe exceeded zero. The reflectivity was measured concurrently by the Wyoming Cloud Radar (WCR) [*Vali et al.*, 1998] onboard the same aircraft. Data from the first uncontaminated radar gate of a side-looking radar beam were used. This gate was 75–90 m displaced horizontally from the 2D-C probe, and the reflectivity data, sampled at ~30 Hz, were averaged along-track to 1 Hz to match the 2D-C data frequency. The

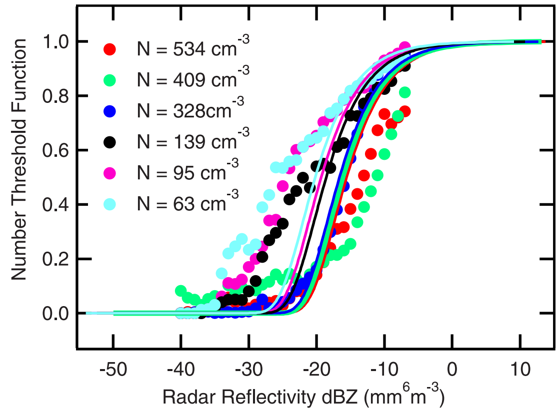

Figure 2. Comparison of the theoretical threshold function to the observational results. The theoretical curves are the same as in Figure 1 but for values of N corresponding with the observed mean values of N. The colors of the theoretical curves correspond to those representing the observational results given in the figure legend. The observations were made off the coast of N. California and Oregon. The averaged droplet concentrations shown in the legend correspond to the cases (denoted by date yymmdd) of 990809, 990816, 990817, 060629, 060613, and 060523, respectively.

resulting reflectivity (Z) values were binned in integer increments, i.e. the bin size is 1 dB. The probability of drizzle at a given value of Z is defined as the number of occurrences in this Z bin *with* drizzle presence as defined above divided by all occurrences in this bin. The probabilities were computed by accumulating occurrences in all Z bins for all flight legs during any flight. The cumulative length of these flight legs varied between 292 and 705 km for the six flights used in this study. Three of the flights were conducted in summer 1999 within 100 km of the Oregon coast [*Wang and Geerts*, 2003]. The three other flights were conducted 260–670 km offshore the South Oregon and North California coasts, in summer 2006. All six flights were conducted during the daytime.

[11] The mean N was derived from the FSSP probe, as an average for the same cumulative flight length, for each flight. Figure 2 compares the values of the number threshold function computed from equations (5a) and (5b) using the values of N for the 6 cases analyzed here, to the observational results from the Wang-Geerts approach. It is clear that the theoretical number threshold function well describes the observational results in general, providing observational support for the theory, and theoretical support for the Wang-Geerts approach. Nevertheless, there are some differences between the theory and the observations, which may be due to the following reasons. First, according to equations (4a) and (4b), the differences may arise from the differences in the spectral shape of the cloud droplet size distribution, which determines the steepness of the transition with x_c. Second, the relative broadness of the observed transition region from non-precipitating to precipitating reflectivities, compared to generally steeper theoretical curves, may also be due to the variability in droplet size distributions. Third, some drizzle drops may escape detection of the 2D-C probe because of its small sampling volume, leading to underestimated drizzle probability. Finally, although the autoconversion process is expected to dominate the threshold behavior of the cloud-to-rain transition, effects from processes such as accretion cannot be completely ruled out (especially for cases with large reflectivities), and these neglected effects may be responsible for the differences as well.

3. Dependence of Threshold Reflectivity on Droplet Concentration N

[12] Previous observational studies have yielded a large range of values for the threshold reflectivity. For example, studies based on in situ measurements of cloud drop size distributions for marine [*Frisch et al.*, 1995a, 1995b; *Baedi et al.*, 2002] and continental [*Baedi et al.*, 2002] clouds have demonstrated the presence of a sharp increase in radar reflectivity associated with drizzle formation over the range from −20 to −15 dBZ. *Sauvageot and Omar* [1987] found a threshold of −15 dBZ for continental stratocumulus clouds. *Frisch et al.* [1995a, 1995b] indicated that radar reflectivities lower than −18 dBZ are usually associated with non-precipitating clouds whereas reflectivities greater than −16 dBZ tend to be correlated with the presence of droplets of diameter $\geq$ 50 μm. *Mace and Sassen* [2000] showed that for continental clouds observed over the ARM SGP site, layers with maximum reflectivity ≥ -20 dBZ nearly always contain drizzle. *Baedi et al.* [2002] showed that the maximum radar reflectivity due to the non-drizzling parts of clouds is around −20 dBZ whereas the minimum reflectivity due to the drizzle component is about −10 dBZ, and on average there is a jump of approximately 10 dBZ in reflectivity between drizzle-free and drizzle-contaminated clouds. *Wang and Geerts* [2003] demonstrated that in marine clouds the threshold varies between −19 and −16 dBZ for three different cases.

[13] What causes the differences in these empirical values of threshold reflectivity? The dependence of the threshold reflectivity on N revealed by our theoretical results shown in Figure 1 provides physical insight with regard to this issue, and warrants further examination.

[14] Although the threshold behavior is clear from Figure 1, the cloud-to-rain transition cannot be characterized as a step function, which leads to some ambiguity in defining the threshold reflectivity. Therefore, we introduce the concept of p-threshold reflectivity, defined as the reflectivity that corresponds to the threshold function T_N = p. With this definition, we can derive the relationship between the p-threshold reflectivity and the droplet concentration as follows. First, according to equation (5a), given T_N = p, we can obtain a corresponding $x_{cp} = x_c(p)$. Then the p-threshold reflectivity is given by

$$Z_{cp} = 7.1 \times 10^{-16} N^{1/2} x_{cp}^{-1}. \quad (6)$$

[15] Expressing it in the unit of dBZ for Z, we have

$$dBZ_{cp}\left(\mathrm{mm^6 m^{-3}}\right) = 10\log\left(10^{12}\right) Z_{cp} \approx -31 + 5\log N - 10\log x_{cp} \quad (7)$$

[16] In radar-related studies, it seems reasonable to consider p = 0.9, which corresponds to x_{cp} = 0.1. Substitution

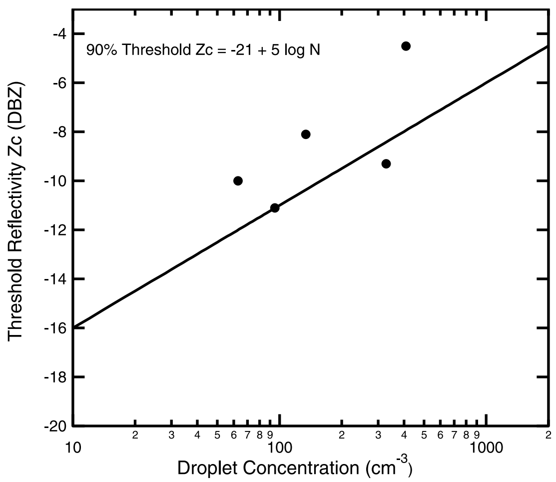

Figure 3. Dependence of threshold reflectivity on droplet concentration. The black line and the dots represent the theoretical and observational results, respectively.

of this value into equation (7) yields the dependence of the 90% threshold reflectivity on N

$$dBZ_{cp}\left(\mathrm{mm}^6\mathrm{m}^{-3}\right) = -21 + 5\log N \qquad (8)$$

[17] Equation (8) reveals that the threshold reflectivity increases with increasing N. From the observational data shown in Figure 2, we determined 5 pairs of N and threshold reflectivity. (The 90% probability threshold reflectivity could not be determined for Case 1, which had the largest value for N and very little drizzle.) Figure 3 compares the theoretical dependence of the 90% threshold reflectivity on N with these observational results. It is evident from the figure that the observational results compare favorably with the theoretical expression, providing observational support for the theoretical formulation. The increase of threshold reflectivity with increasing N is consistent with the notion that clouds with more droplets can hold more cloud water [*Berg et al.*, 2006], and indicates that the differences in the empirical values of threshold reflectivity reported in literature likely arise from the differences in N between the corresponding clouds examined. The increase of threshold reflectivity with increasing N stems primarily from that a higher N leads to a smaller mean radius but a larger r_c of the droplet population [*McGraw and Liu*, 2003, 2004; *Liu et al.*, 2004]. A larger r_c indicates that on average larger droplets are needed to activate the collection process.

[18] It is noteworthy that the theoretical formulation also suggests other possible reasons for the differences in empirical threshold reflectivity. For example, different researchers might have used different criteria for defining the threshold reflectivity (e.g., different p values), and clouds in question might have different spectral shapes of the cloud droplet size distribution. These issues will be addressed elsewhere.

4. Concluding Remarks

[19] The theoretical threshold function previously derived for representing the autoconversion process in atmospheric models is related to radar reflectivity. The new formulation clearly shows a general sharp transition when radar reflectivity exceeds some threshold value, and compares favorably with observations collected from marine stratiform clouds over a wide range of conditions. A simple relationship is derived between the threshold reflectivity and the droplet concentration, revealing that the threshold reflectivity increases when droplet concentration increases; this relationship compares favorably with observations. The dependency of the threshold reflectivity on droplet concentration provides a physical explanation for the wide range of values that have been empirically obtained for this quantity. The theoretical formulation also suggests other possible reasons for the differences in empirical threshold reflectivity derived in various studies. For example, different researchers might have used different criteria for defining the threshold reflectivity (e.g., different p values), and the clouds in question might have different spectral shapes of the cloud droplet size distribution. Furthermore, only the dependence on droplet concentration is discussed in this paper because of its close link to the Wang-Geerts approach. If the threshold function is defined with respect to other quantities such as the liquid water content, different results are expected. All these suggest the necessity to specify the criteria and the approach used in the empirical determination of threshold reflectivity.

[20] The following three points are noted in passing. First, although the focus of this paper is radar reflectivity, the agreement between the theoretical formulation and observational results provides additional observational validation of the theoretical autoconversion parameterization we have presented previously. Second, the effect of spectral shape of the cloud droplet size distribution on the threshold behavior and relationship between threshold reflectivity and droplet concentration is ignored at present; we plan to examine this issue in detail when additional data become available. Finally, the theoretical formulation has many potential applications. For example, the theoretical relationship of the threshold reflectivity to the droplet concentration may be applied to cloud radar observations of the 2nd aerosol indirect effect, and its inverse may also be used to infer the droplet concentration from radar measurements, which of course is limited by the accuracy of the measurement of radar reflectivity.

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P. Daum, Y. Liu, M. Miller, and R. McGraw, Brookhaven National Laboratory, Atmospheric Sciences Division, Bldg. 815E, Upton, NY 11973–5000, USA. (lyg@bnl.gov)

B. Geerts, Department of Atmospheric Sciences, University of Wyoming, Laramie, WY 82071, USA.