ATSC 5010 – Physical Meteorology I Lab
Lab 4 – saturation vapor pressures and moisture variables

➔ How does saturated vapor pressure change with temperature? As we saw in lecture, we can derive a relationship considering the gibb's function remains constant during a phase change (during phase equilibrium, two phases are in equilibrium with each other). We arrive at the following equation:

$$\frac{de_s}{dT} = \frac{L}{T(\alpha_2 - \alpha_1)}$$

where:
e_s is saturation vapor pressure over liquid water or ice
α is the specific volume of the vapor and the liquid water or ice
L is the amount of heating (or cooling) for transition between vapor and liquid or vapor and ice.

For transition between vapor and liquid in normal atmospheric conditions, $\alpha_2 \gg \alpha_1$ and substituting assuming an ideal gas results in the familiar form of the C-C equation:

(1a) $\dfrac{de_{sw}}{dT} = \dfrac{L_{lv}e_{sw}}{R_v T^2}$

or for a transition between vapor and ice:

(1b) $\dfrac{de_{si}}{dT} = \dfrac{L_{iv}e_{si}}{R_v T^2}$

➔ To a first-order approximation, we often assume $L=L_0$ (latent heat at the triple point) and is independent of temperature. We then integrate (1a) or (1b) to solve for $e_s(T)$:

(2): $\ln\left(\dfrac{e_s(T)}{e_{s0}}\right) = \dfrac{L_0}{R_v}\left(\dfrac{1}{T_0} - \dfrac{1}{T}\right)$

➔ We will learn in class that L is not constant with temperature—we will leave as an exercise for homework the consideration of $L(T)$ and a significantly more complicated formula relating e_s and T.

For now we will consider an empirical function for e_s on temperature. Here we are ignoring the solution to C-C and computing directly e_{sw} (and e_{si}) as a function of temperature. (While we wouldn't want to do this by hand, doing it on a computer is easy….)

For saturation vapor pressure over liquid (note this is natural log, not LOG_{10}):

$$
\begin{aligned}
LOG(e_{sw}) &= -\frac{0.58002206 * 10^4}{T} + 0.13914993 * 10^1 - (0.48640239 * 10^{-1})T \\
&+ (0.41764768 * 10^{-4})T^2 - (0.14452093 * 10^{-7})T^3 \\
&+ (0.65459673 * 10^1) * LOG(T)
\end{aligned}
$$

For saturation vapor pressure over ice (note this is natural log, not LOG_{10}):

$$
\begin{aligned}
LOG(e_{si}) &= -\frac{0.56745359 * 10^4}{T} + 0.6392527 * 10^1 - (0.96778430 * 10^{-2})T \\
&+ (0.62215701 * 10^{-6})T^2 + (0.20747825 * 10^{-8})T^3 - (0.94840240 \\
&* 10^{-12})T^4 + (0.41635019 * 10^1) * LOG(T)
\end{aligned}
$$

Exercise:

1. Assuming no variation of L on temperature (ie L_{lv} is constant and L_{iv} is constant), solve (2) for $e_{sw}(T)$ and for $e_{si}(T)$

2. Build a function called *sat_vapor* that takes an vector input of temperatures in Kelvin and returns a corresponding vector of saturation vapor pressures in Pascal. Allow your function to have keywords: ice, liquid, Lconst, polynomial. Your function should have the following behavior:

   a. If keyword 'ice' is set, the saturation vapor pressure over ice is computed.

   b. If keyword 'liquid' is set, the saturation vapor pressure over liquid is computed.

   c. If keyword 'Lconst' is set use the approximation that L is constant over the temperature range to compute saturation vapor pressure.

   d. If keyword 'polynomial' is set, use the polynomial fit to compute directly the saturation vapor pressure.

   Note: when the function is called, it should be called with 2 keywords set: first keyword set should be 'ice' or 'liquid' (never both!!) and the second keyword set should be 'Lconst' or 'polynomial' (again, never both!!)

3. Build a procedure (atsc5010_*yourname*_lab4) that produces a vector of temperatures from -40 to +100 C with 0.1 C resolution. In this procedure:

a. Call *sat_vapor* several times, each time with the appropriate keyword(s) set to compute saturation vapor pressure over ice and/or over liquid for the temperature range from -40 to +100 C. You will call it 4 times:

   i. 'liquid' and 'Lconst'

   ii. 'liquid' and 'polynomial'

   iii. 'ice' and 'Lconst'

   iv. 'ice' and 'polynomial'

b. Plot a graph (P-T diagram) showing the saturation vapor pressure over liquid and ice from -40 C to 100 C. Use red for 'Lconstant' and blue for 'polynomial'. For saturation vapor pressure over liquid at temperatures below 0 C, use a dashed line. All other lines should be solid. Consider that portion of the atmosphere for which we (as atmospheric scientists) are interested. Make the same plot as above, but focus in on the temperature range from -40 to 10 C, constraining the vapor pressure range from 0 to 1000 Pa. For both plots in (b), put them side-by-side in the same plotting window.

For plots in (c) and (d) (a total of 4 plots), use the same plot window, such that (c) will be the top two plots in the window and (d) will be the bottom two plots. You will need to use the LAYOUT and CURRENT keywords in you plot function.

c. Consider the comparison between 'Lconstant' and 'polynomial'. For both saturation vapor pressures over liquid, plot the difference between the computed values using 'Lconstant' and 'polynomial' as a function of temperature (for temperature range -10 to + 15C). Do the same for saturation vapor pressure over ice (temperature range -30 to 0 C).

d. Similar to (c), Plot the percent difference between the two computations as a function of temperature (both for saturation vapor pressure over liquid and saturation vapor pressure over ice for the same temperature ranges in (c)).

e. Now consider our most accurate model, 'polynomial', only. Noting that relative humidity wrt liquid, h, is defined as e/e_{sw} and relative humidity wrt ice, h_i, is defined as e/e_{si}. Consider a cloud that is composed of liquid water and ice crystals (mixed phase cloud). If the cloud is just saturated wrt to liquid, calculate (and plot) the relative humidity wrt ice as a function of temperature.

Remember to use axis labels and titles on all your plots.