ATSC 5010 – Physical Meteorology I Lab
Lab 9 – Daily Average Solar Flux

We learned in Chapter 2 of Petty that solar flux incident on the earth depends not only on the output (Radiance/Intensity) of the sun, but also on the distance from the earth to the sun. If we ignore slight variations in both of the above, then we can compute a *Solar Constant,* S_0, that represents a more or less constant flux of solar radiation at the top of the atmosphere equal to 1370 W m^{-2}.

If we now want to compute the flux density (per unit area) at any location at the top of the atmosphere, we need to consider the zenith angle between the sun and that location—that is the angle between a vector normal to our surface and a vector pointing towards the sun. A zenith angle of 0 implies the sun is directly overhead. This angle depends on three parameters: (1) latitude—generally at higher latitudes the zenith angle increases, (2) the time of day—at solar noon the zenith angle passes through a minimum, at dawn and dusk the zenith angle is 90 deg, and (3) the day of year through the dependence on declination—related to the 'tilt' of the earth on its axis.

Simple cases are easily visualized. For instance, during the equinoxes, at the equator, the zenith angle begins at 90 deg at dawn, passes through 0 deg at noon (directly overhead), and returns to 90 degrees at dusk. At a latitude of 30 deg, the minimum zenith angle (during the equinoxes) is 30 deg, similar for 45 deg lat, 60 deg lat, etc.

During the solstices, one needs to consider the declination (tilt). During the summer solstice (in the northern hemisphere) at a latitude of 45 degN, the minimum zenith angle at solar noon is 45-23.5 = 21.5 deg. At the northpole, the zenith angle is constant 66.5 deg (90 – 23.5). Between the solstices and the equinoxes, computation of the angles becomes more complicated—but for any given declination angle the MINIMUM zenith angle (that at solar noon) is equal to the latitude *minus* the declination angle.

Following the above discussion, a logical question is 'How does the declination angle change over the year?' We can construct a simple function that determines this based on the DOY (day of year). Given 365 days in a year:

1. Construct a vector with 366 values ranging from 0 to 2π (note day 0 and day 365 are the same)
2. Shift the function 10 pts to the left (this accounts for the winter solstice occurring 10 days before the end of the year)
3. Take the COSINE of this vector, resulting in a sin/cos function with one period over the full 365 days
4. Multiply your function by -23.5 (the solar declination in degrees in the northern hemisphere during the winter solstice).
5. If you plot solar declination as a function of DOY, it should be obvious what days represent the equinoxes and what days represent the solstices.

Combining now the specific results from earlier and the general form for our solar declination angle, we can easily calculate the *minimum* zenith angle for any latitude for any day of the year:

$$\Theta = \phi - \delta \quad \textit{where } \phi \textit{ is latitude and } \delta \textit{ is declination}$$

and this can be built in the form of an n x m matrix (n is latitude ranging from -90 to +90 and m is DOY).

If we only wanted to compute the *maximum* flux the problem is essentially complete. Given the minimum zenith angle, the maximum flux is simply:

$$F = S_o * \cos(\Theta)$$

BUT….we are interested in determining the average flux over the course of a 24 hour period. To do that, we need to consider how the zenith angle changes with time of day AND the length of a day. The calculation of both of these requires the introduction of the *hour_angle*. An hour angle is a *unit of time measured in degrees*. The earth rotates 15 deg/hour. So an hour angle, τ, of 45 degrees represents 3 hours. The hour angle can be calculated from the latitude and the declination:

$$\tau = 2 * \mathrm{acos}(m) \quad \textit{where } m=(-\tan(\phi)*\tan(\delta))$$

In the above definition, τ is only defined for m between -1 and +1. However, for large absolute(ϕ) (i.e. high latitudes), it is possible for m to exceed -1 and +1. In these cases one must manually set m to a proper value.

The number of hours in a day is easily computed from the hour angle through the following:

$$\text{n\_hours} = \tau/15$$

The zenith angle can now be determined for a given latitude and DOY, throughout the course of the day (as the sun moves across the sky). First consider a day that has an hour angle of 210 degrees (a 14 hour day). Consider the angle that is associated with the zenith – solar noon. For this case there exist 105 degrees before noon and 105 degrees after noon. So we can break this up into hour angles ranging from -105 degrees at dawn to +105 degrees at dusk. This corresponds to an angle of 0 at solar noon. In this framework, the zenith angle is given by the following:

$\Theta = \arccos\{ \sin(\phi)\sin(\delta) + \cos(\phi)\cos(\delta)\cos(\tau) \}$ where τ varies throughout the day (between -105 and +105 and is 0 at solar noon in the above example).

Now we can compute the daily *average* solar flux for a given day and latitude by summing:

$$F_0 = S_0 * \Sigma(\cos(\Theta)\Delta t)/24$$

You could just as easily replace delta_t with delta_hourangle and divide by 2π.

Exercise:

1. Using IDL create a vector that represents latitude ranging from -90 to +90 with a resolution of at least 1 degree. Compute the MAXIMUM solar flux (this is the solar flux at solar noon) as a function of latitude assuming a declination of 0 degrees. This is the flux you would expect during the equinoxes.
2. Now, compute the MAXIMUM solar flux as a function of latitude for both the winter solstice and the summer solstice.
3. Plot the above three functions on the same graph. Label each line. To ease in interpretation, plot latitude on the y-axis and allow it to range from -90 to +90 with tick intervals of 30 degrees.

    a. *If you integrate the above functions from -90 to + 90 degree in latitude, the result is largest for the equinoxes. Does this then imply that at solar noon more radiation strikes the earth during the equinoxes then during the solstices? Explain this apparent discrepancy.*

4. Construct an n x m matrix (where n is latitude from -90 to +90; and m is days from 0 to 365) that represents the number of hour angles in a day. The result should be total number of degrees (or radians) in that day. From that compute a second n x m matrix that represents the number of hours in a day.
5. Plot the number of hours in a day as a function of DOY for the following latitudes: 0, 20, 40, 60, and 80 degN on the same graph. Label the lines. Note, all of the lines should have values for all points. The vector for 80 degN should contain many values that are either 0 or 24—but you will need to force those values!
6. Estimate the DOY for each of the equinoxes and for the summer and winter solstice. Plot an x-y graph that shows the length of day vs latitude for each of the 4 days on the same graph. Label the lines. Plot latitude on the Y-axis and allow it to range from -90 to + 90 with tick intervals of 30 degrees.
7. Contour length of day as a function of latitude (on the y-axis) and DOY (on the x-axis). Pick one color and contour all the lines in that color. Draw contour lines for 1, 6, 8, 10, 12, 14, 16, 18, and 23 hours. Approximate with thick black vertical lines the approximate days for the equinoxes and the solstices.

    a. *Compare your graph with the (hopefully) similar one found at the end of this lab. Describe differences between the two. The plot at the back of the lab is more accurate—it better represents the actual length of day for a given location and DOY. What simplifications or assumptions did we make in our derivation that may have led to differences we see in the plots?*

8. Compute the zenith angle. Zenith angle depends on latitude, DOY and time of day, thus you will need to represent this as an n x m x p matrix. n and m will represent latitude and DOY as before. **p will represent some arbitrary fraction of the day.** In this exercise we are going to split our daylight into equal fractions of 100 (thus p will have 100 values). Thus p will have 101 points from 0 to 100. The hour angle will represent steps of 1/100th

of the total hour angles *for that day*. For a 12 hour day (180 hour angle degrees), p0 will be -90, p1 will be -88.2, p3 will be -86.4,…p50 will be 0, p101 will be +90. In this context, p50 will *always* be 0 (and hence represents solar noon). It is this value that will lead to the minimum zenith angle for a given day. Likewise, for a 15 hour day (225 hour angle degrees), p0 will be -112.5, p2 is -110.25, p50 is 0, p100 is +112.5.
So now, for each latitude and for each DOY you should have a 101-pt vector that ranges over the full range of hour angles for the length of that day. (NOTE – these 101 pts represent a *time step*, however, the time step is not the same when comparing a given latitude/DOY pair with another….The amount of the time step depends on the length of day. For shorter days, the time step is shorter because you are dividing a shorter number of hours (or smaller number of hour angles) by the same divisor, 100).

9. Now you can use you new matrix to compute the daily average flux density for each latitude and DOY (this should result in an n x m matrix). Essentially you are numerically integrating the flux density across the day (the 'p' part of the matrix that you computed in the last step).
10. Contour the daily average flux density for latitude (y-axis from -90 to +90) and DOY of year. Contour every 25 W/m^2 beginning at zero and label every 4 line (0, 100, 200, …). Overplot the declination angle as a function of DOY.

    a. *At what latitude is the greatest daily average flux density (insolation) found? Provide an explanation for why it is found at this latitude.*
    b. *Consider the daily average flux density integrated over a year. Estimate latitude at which the greatest value of this is found. Provide an explanation why it is found at this latitude.*