Co-location of aircraft and radar data – II.

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The focus of this short report is an assessment of the precision with which King Air aircraft and SPol radar data can be co-located in space. The question is more general, of course, but no attempt is made here at a comprehensive study.

The problem is obvious: Since aircraft and radar data are generated in different position reference systems, some conversion is needed to a common system that allows ready determination of aircraft position (and the in situ and remote sensing data from that platform) with respect to the array of radar pixels. The most convenient choice for this, from the points of view of ease of plotting and intuitive appreciation, is to use the East-West, ***x***, and North-South, ***y***, distances from the radar site to a target point (a radar pixel or the aircraft position), plus altitude, ***z.***

Aircraft position from the GPS is given as latitude and longitude, $\mathbf{[\mathit{lat,lon}]_{\mathit{ac}}}$. These data are smoothed with IRS data to eliminate interruptions or glitches in the GPS signal, but the GPS data provides the basic accuracy of the position. That accuracy can be as good as a few decimeters with good reception from multiple GPS satellites and GPS ground receivers, and with special post-flight processing.

The native coordinates for radar data are ***range (R), azimuth (A)*** and ***elevation angle (E)*** referenced to the location of the radar, $\mathbf{[\mathit{lat,lon}]_{\mathit{SPol}}}$, in this case SPol on Barbuda. Without tracking back to details, we assume that the geographic position and the absolute azimuthal orientation of the radar are known with adequate precision.

The conversion from **[*R,A,E*]** to **[*x,y,z*]** is the well-known spherical to Cartesian conversion that can be carried out to any desired precision. Refraction of the beam is neglected here, that is, a correction for that is assumed to have been done, if needed, and already included in the **[*R,A,E*]** values. Both the **[*R,A,E*]** or **[*x,y,z*]** values are with respect to a tangent plane to the Earth at the location of the radar.

The main issue is how to convert latitude and longitude data to distance and direction for the radar site, since this involves taking into account the shape of the Earth. Because of the ellipsoidal shape of the Earth, the calculation is not straightforward[1] and there are some

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[1] Latitude values usually quoted, including the GPS-derived positions, are geodetic latitudes, not geocentric latitudes. Geodetic latitude is defined by the intersection of the perpendicular to the local tangent with the equatorial plane. At 17 degree latitude the difference between the two values is about 0.1 degrees. See, for example, "Aerospace Coordinate Systems and Transformations" by G. Minkler and J. Minkler, Magellan Book Co., Baltimore, MD. 1990.

fundamental limitations to comparing positions on a curved surface with representations on a plane. Two of the many different approaches to the problem are of relevance here: the azimuthal equidistant and the ECEF/ENU (Earth-Centered-Earth-Fixed to East-North-Up) transformations described in following sections. We limit the analyses here to points directly on the surface, $\boldsymbol{z} = 0$, and for radar pixels $\boldsymbol{E} = 0$, and $\boldsymbol{R}$ replaced with its projection, $\boldsymbol{R_g}$, onto the tangent plane.

A. Azimuthal Equidistant Projection (AEP).

The basis of this method is to derive, for two points defined by their **[*lat,lon*]** coordinates, the great-circle distance ***d*** along the Earth's surface between the two points and the bearing $\boldsymbol{\psi}$ of the line connecting the two points with respect to geographic North, yielding $[\boldsymbol{d},\boldsymbol{\psi}]$. The algorithm developed by Vicenty[2] for carrying out this calculation uses an iterative procedure in order to achieve precision claimed to be on the order of millimeters. The distances and angle so derived are the basis of the so-called azimuthal equidistant projection of the curved Earth surface onto a plane. That plane can be the tangent plane at the radar site, and then this projection is well matched to the native radar coordinate system.

Thus, the first step in the transformation is $[\,[lat,lon]_{SPol} - [lat,lon]_{ac}\,] \Rightarrow [d,\psi]_{ac}$; we'll return later to how this is done in practice.

The next step, $[d,\ \psi]_{ac} \rightarrow [x,\ y]_{ac}$, with $\boldsymbol{z}$=0, even though it appear to be the same as the $[R_g,\ A] \rightarrow [x,\ y]$ polar to Cartesian transformation, has a fundamentally different character. The numerical results are very close for moderate distances, but it is still worth considering the concepts involved. That concept is well illustrated in the figure on the next page, taken from a National Geographic Society publication[3] (see explanations within the red rectangle and ellipse). The point is that while radial distances and angles are preserved in the projection, distances along lines different from the radials, hence also the ***x*** and ***y*** distances in general, are increased due to the stretching of the curved surface to the plane. This is also expressed by the Tissot indicatrix diagram[4] for this projection: circles become ellipses of increasing aspect ratios with increasing distance from the center, with the short axes of the ellipses lying along radials. Thus, $\boldsymbol{x} = \boldsymbol{d} \cdot \cos\psi$ and $\boldsymbol{y} = \boldsymbol{d} \cdot \sin\psi$ yield values that are not the actual distances we seek in the tangent plane. As a consequence, aircraft tracks converted

[2] Vicenty, T, 1975: Direct and inverse solutions of geodesics on the ellipsoid with application of nested equations. *Survey Review*, **22**, 88-93.

[3] "The Round Earth on Flat Paper", National Geographic Society, 1950, 126 pp.

[4] See: Snyder, J. P., 1997: Map Projections – a working manual. USGS Professional Paper 1395, 383 pp. and http://www.3dsoftware.com/Cartography/USGS/MapProjections/

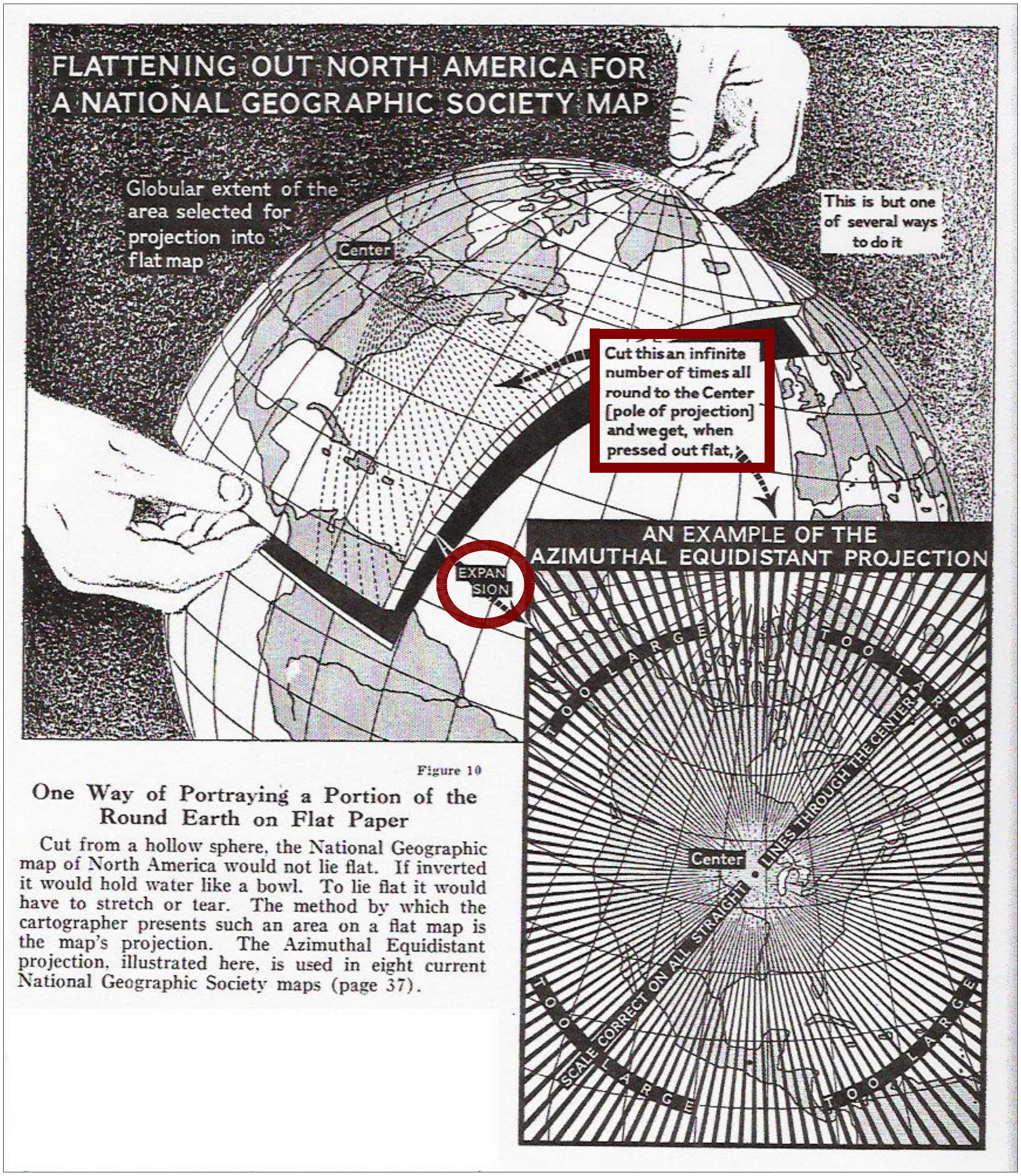

Figure 10

One Way of Portraying a Portion of the Round Earth on Flat Paper

Cut from a hollow sphere, the National Geographic map of North America would not lie flat. If inverted it would hold water like a bowl. To lie flat it would have to stretch or tear. The method by which the cartographer presents such an area on a flat map is the map's projection. The Azimuthal Equidistant projection, illustrated here, is used in eight current National Geographic Society maps (page 37).

from **[*lat,lon*]** to **[*d,ψ*]** using the Vicenty transforms, and then to **[*x,y*]** via $x = d \cdot \cos\psi$ and $y = d \cdot \sin\psi$ will not be perfectly accurate in shape.

Computations:

1) A MATLAB implementation of the Vicenty algorithm, **vdist.m**, is available via the Mathworks website[5]. An IDL transplant of this code **vdist.pro** was also generated

[5] http://www.mathworks.com/matlabcentral/fileexchange/loadFile.do?objectId=5379&objectType=file Part of the code was un-commented, and the steps of getting azimuth to be in the right quadrant corrected, for use in

for this work and is available from the authors.

2) The MATLAB routine **eqaazim.m** (part of the Matlab mapping toolbox) also solves the **[*lat,lon*]** to **[*d,ψ*]** transformation, and yields results nearly identical to the **vdist** procedure, as shown in Fig. 1. Contour lines are in meters. Maximum differences are < 2 meters.

3) The Vicenty algorithm is also the basis of the utilities available via a National Geodetic Survey web page[6], where on-line calculations can be performed, and both Fortran and PC routines can be downloaded. Comparison of output from the Fortran version of the **invers3d** routine with results from **vdist.pro** yielded maximum differences of $\Delta d \approx 7$ meters and $\Delta\psi \approx 0.004°$, over 100-km distances. These translate to maximum **x** and **y** differences of < 10 meters.

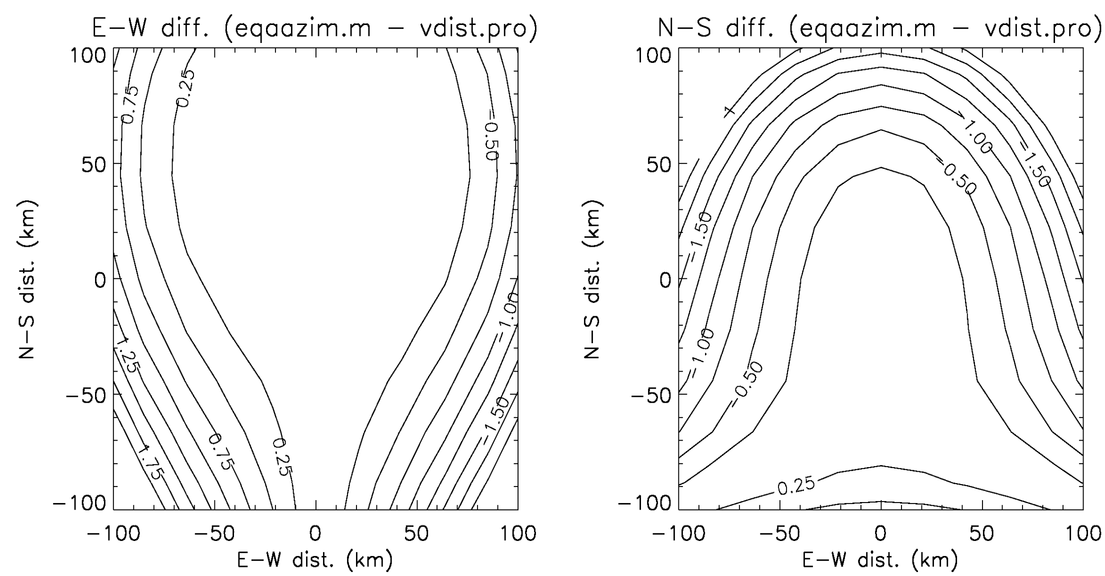

Fig. 1 Comparison of x (E-W) and y (N-S) distances derived by two AEP algorithms for points distributed in a 200x200 km domain. Contours are in meters.

this comparison, so someone wishing to use this code should contact the authors to obtain a copy of the code.

[6] http://www.ngs.noaa.gov/TOOLS/Inv_Fwd/Inv_Fwd.html

B. ECEF/ENU transformation

The solo-ii radar analysis and display routine (NCAR – EOL) has built-in features for converting radar pixel locations to **[*lat,lon,z*]** and to **[*x,y,z*]** using the ECEF/ENU point transformation[7]. The ECEF frame has its origin at the earth's center and the basis vectors point along the axis of rotation and two perpendiculars to that. The ENU frame is centered at some point on or above the earths surface and point true east, north and up. The transformation assumes a spherical surface with radius computed from the oblate ellipsoid shape to extend from the Earth center to the center of the ENU frame (the radar site or the aircraft location, depending on which way the transformation is being carried out). These assumptions in solo-ii imply the use of geocentric latitudes. The X-Y plane of the ENU frame is not coincident with the tangent plane of the oblate spheroid.

In order to examine the results from the solo-ii code, IDL versions were created from the C routine, called **latlon2xy.pro** and **xy2latlon.pro.** Results from this procedure differ significantly from those derived via the azimuthal equidistant projection, as shown in Fig. 2.

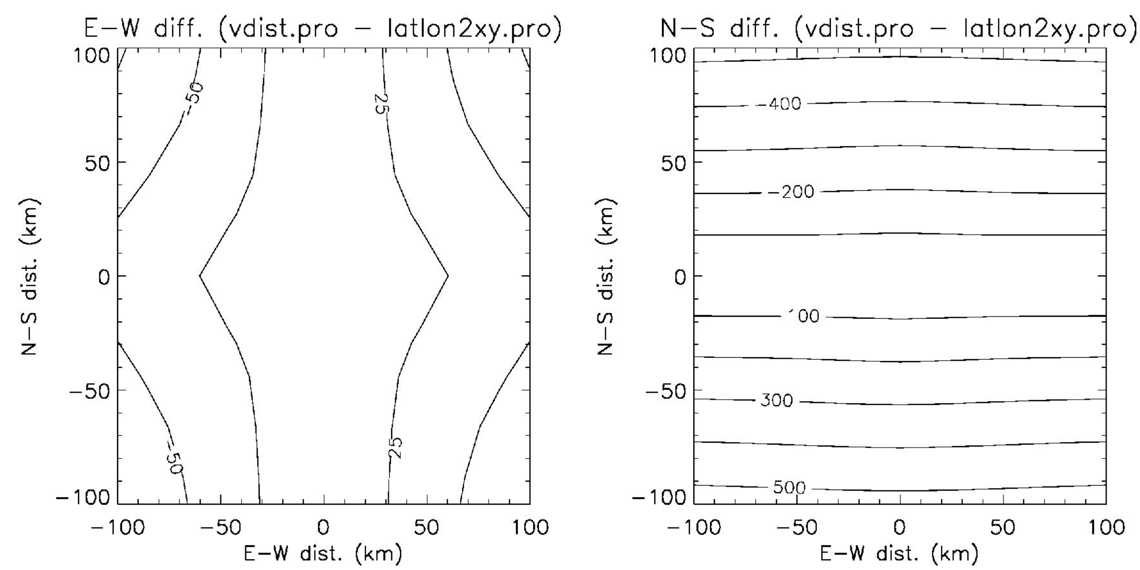

Fig.2 Difference in x and y distances between AEP and ECEF/ENU projections. Contours are in meters.

Even the East-West differences are more than ten times larger than those between the two types of azimuthal equidistant algorithms discussed earlier. The North-South differences are yet larger, reaching values of 500 m for points about 100 km N or S of the radar. These large differences are almost certainly due to the different orientation of the projection planes for

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[7] "Aerospace Coordinate Systems and Transformations" by G. Minkler and J. Minkler, Magellan Book Co., Baltimore, MD. 1990

the two reference frames and to the difference between using geocentric and geodetic latitudes.

C. Radar 'hits' on aircraft.

Radar 'hits' i.e. anomalously high reflectivity values in some radar pixels due to the aircraft being in that location, can serve to check the degree of agreement between the positions reported by the two systems: the SPol radar via solo-ii, and the aircraft GPS.

At first, two such event are used, from the King Air flight on 20041213. GPS estimates of position are from an older Trible VeeSix receiver with accuracies of ~100 m. Finding the events was relatively easy: reflectivity values in a few pixels about 20 dBZ over the rest of the small cloud constituted pretty good evidence. That could be further confirmed by the proximity of the aircraft from its track data. During one pass, two pixels had excessive reflectivities, in the second pass there were four such pixels. The events were at about 70 km from the radar site so they provide a difficult test case for the position comparison but are limited at the same time by the radar volume resolution. At closer range the resolution would be better, but more radar pixels will have excessive reflectivities and provide the same sort of ambiguities in the end.

The table on the next page shows as 'measured' values the GPS position of the aircraft, $\mathbf{[\mathit{lat,lon}]_{ac}}$, and the position of the target pixel, $\mathbf{[\mathit{x,y}]_{tp}}$. The latter values were read from the solo-ii display (via 'examine' and 'data widget' options). The conversion used to get these values from $\mathbf{[\mathit{R}_g, \mathit{A}]_{tp}}$, namely $x=R_g\sin A$ and $y=R_g\cos A$, was found to be identical in solo-ii and in manual calculations for a number of points within the whole radar domain. The $\mathbf{[\mathit{lat,lon}]_{tp}}$ values read from solo-ii also agree with the values obtained from **xy2latlon.pro**.

Since the calculation of $\mathbf{[\mathit{x,y}]_{ac}}$ from $\mathbf{[\mathit{lat,lon}]_{ac}}$ via the ECEF/ENU transformation (latlon2xy.pro), and the derivation of $\mathbf{[\mathit{x,y}]_{tp}}$ values in the solo-ii software are based on the same algorithms, we expected good agreement between the values in the boxes shaded light green and yellow in the table. Yet, differences up to 100 m in **x** and up to 500 m in **y** are seen. Even though the AEP algorithm (vdist.pro) is further from the solo-ii method in its basis, the results in the light gray boxes are closer to the solo-ii measurements than those in the light yellow boxes. The same conclusion is reached looking at the differences in terms of latitude and longitude.

Another set of seven separate 'hits' were evaluated for the King Air flight of 20050111 during the period 18:30 to 18:40. The GPS data from this flight were from an Ashtech Z-Sensor which were re-precessed using carrier phase tracking and static ground

Comparison of position data for King Air 'hits' during 20041213 flight

		MEASURED	CALCULATED		
			xy2latlon.pro	latlon2xy.pro	vdist.pro
20:37:51 UTC		**KA GPS (note 1)**			
	lat	18.1442°			
	lon	-61.97298°			
	x (km)			-15.73	**-15.74**
	y (km)			59.74	**59.41**
		SPol via solo-ii (note 2)			
	lat		18.1398°		
	lon		-61.9720°		
	x ± 0.84 (note 4)	**-15.62**			-15.63
	y ± 0.29	**59.25**			58.92
20:39:46 UTC		**KA GPS (note 1)**			
	lat	18.1454°			
	lon	-61.9754°			
	x			-15.98	**-15.99**
	y			59.87	**59.54**
		SPol via solo-ii (note2)			
	lat		18.1410°		
	lon		-61.9750°		
	x ± 0.86	**-15.94**			-15.95
	y ± 0.36	**59.38**			59.06

note 1: King Air GPS data taken from 1-Hz file, reading a single value without smoothing

note 2: for hit at 20:37:51, using data from one of the two (the Eastern one) pixels with equal reflectivity (36 dBZ)
for hit at 20:39:46, using center of four adjacent pixels (33, 37, 41, 46 dBZ)

note 3: using the difference between the GPS and SPol values via 'ear2xy'

note 4: radar pixel size is 0.8° in azimuth and 0.15 km in range

station differential techniques. Accuracies are better than 1 m, so in that regard this promised to be a better comparison than the 20041213 case. On the other hand, the King Air was only about 35 km from SPol so that discrepancies in results from different algorithms were expected to be harder to detect.

Values of $[x,y]_{ac}$ from $[lat,lon]_{ac}$ from the **vdist.pro** and **latlon2xy.pro** algorithms differed by ~25 m in ***x*** and by ~45 m in ***y*** for the seven events, in accord with Fig.1. These values are indicated by the orange squares in Fig. 3. The difference between aircraft positions from the radar hits (as read off the solo-ii display) and from the aircraft GPS data are shown as blue diamonds in Fig. 3. Two bad points in this set, the first and the fourth, are most likely due to misidentification of the aircraft on the radar image. The second point shows the best agreement, while the remainder indicate discrepancies of up to about 200 m. Again, since the differences of several hundred meters are greater than what can be expected to result from the different transformation algorithms, the identification of 'hits' is the most likely error source.

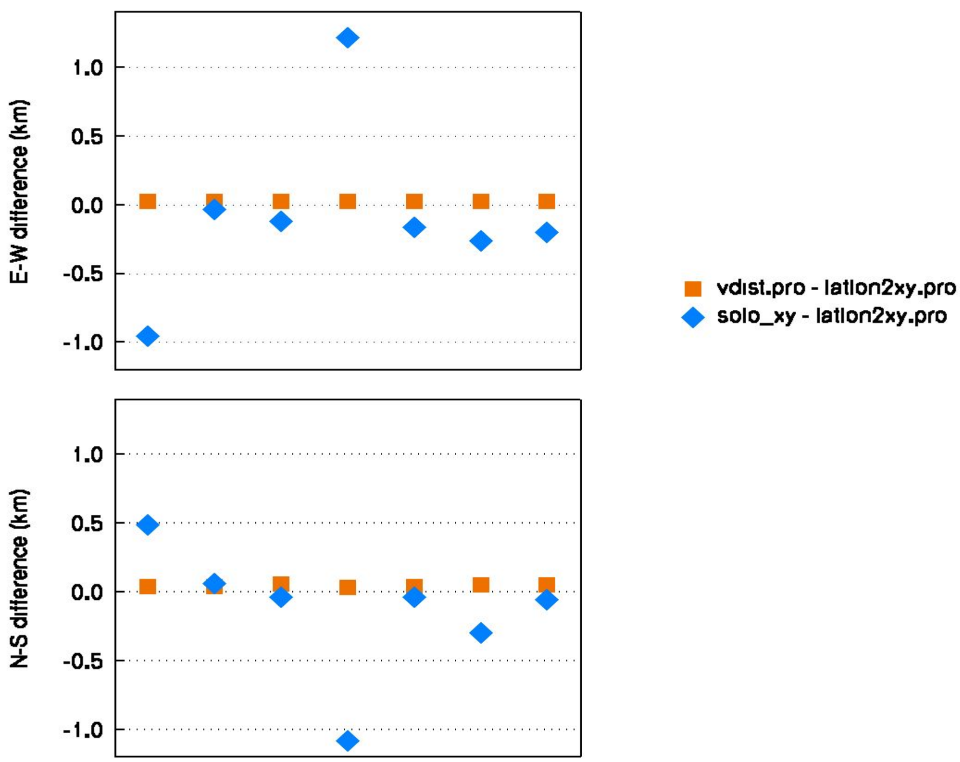

Fig. 3. Differences in ***x*** *(E-W) and* ***y*** *(N-S) for seven radar 'hits' of the King Air on 20050111. The King Air was located roughly 35 km to the E and 10 km to the S of SPol during the period referred to in these data. The orange squares show the difference between the AEP and ECEF/ENU algorithms; these errors are small for such a small separation between the radar and the aircraft. The blue diamonds show the differences (in km)*

D. Conclusions.

1. Locations of pixels in the radar data (SPol) are reported in the solo-ii display (very likely, others as well) in an ENU coordinate system, with the pixels projected onto a plane perpendicular to the radius drawn to the radar site from the center of the Earth. The native measurements of $[\boldsymbol{R_g}, \boldsymbol{A}]_{\mathbf{tp}}$ are converted to $[\boldsymbol{x,y}]_{\mathbf{tp}}$ using simple geometry. The derived quantities $[\boldsymbol{lat,lon}]_{\mathbf{tp}}$ are obtained via an ECEF/ENU transformation algorithm. This transformation incorporates a number of simplifying

assumptions, so that the **[*lat,lon*]$_{\mathbf{ac}}$** values from GPS are not directly comparable. The magnitudes of the differences between the two systems can reach up to 0.5 km at distance of about 100 km to the N or S of the radar (Fig. 2).

2. The best approach to co-locating aircraft position and radar pixels appears to be the conversion of aircraft position from the **[*lat,lon*]$_{\mathbf{ac}}$** to **[*x,y*]$_{\mathbf{ac}}$**, using the AEP methods and to match this to **[*x,y*]$_{\mathbf{tp}}$** from $x=R_{\mathbf{g}}\sin A$ and $y=R_{\mathbf{g}}\cos A$ (from solo-ii or any other software). Comparisons of the three different methods (via **eqaazim**, **vdist** and **invers3d**) of conversion of aircraft coordinates show discrepancies of less than 10 meters for distances up to 100 km from the radar.

3. The accuracy achieved via the method just described is limited by a number of factors. First, there is the inherent inaccuracy of projecting measurements on a curved surface onto a plane. Secondly, radar pixel sizes are several tens of meters (depending on range). The accuracy of the GPS aircraft position is limited, in general, to about the same order of magnitude. Additional uncertainties arise from radar pointing errors – we have no estimates for this. And, as mentioned earlier, diffraction of the radar beam is a further complicating factor, though this can be accounted for, in principle. The evaluation of radar 'hits' failed to give a good estimate of practical accuracies that can be achieved, though the best samples came close to the limit expected from the coordinate conversion. In general, errors increase with distance from the radar, and precision better than 100 m can only be hoped for at radar-aircraft separations of <50 km.

4. Constraining the aircraft data time series of hydrometeor measurements to SPol echo boundaries, for short passes across clouds, may give the best relative comparison between aircraft and radar data.